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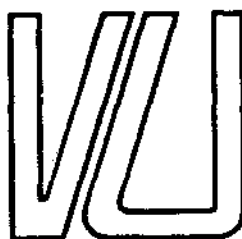
STOCHASTIC MARKET EQUILIBRIA WITH EFFICIENT RATIONING

with an application to the Dutch housing market

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**VRIJE UNIVERSITEIT
FACULTEIT DER ECONOMISCHE WETENSCHAPPEN
EN ECONOMETRIE
AMSTERDAM**

Stochastic Market Equilibria with Efficient Rationing, with an Application to the Dutch Housing Market.

Abstract

In this paper we study a housing market with fixed supplies and demands that have been derived on the basis of discrete choice models. We assume that prices are fixed by the government for policy reasons and that there is excess demand. The population is initially distributed in some exogenously given way over the housing stock and has to be redistributed in accord with its own desires. This can be done by rationing and we are concerned with the question whether there exists some efficient way of rationing, i.e. one in which no possibilities for voluntary movements are left. It turns out that this is the case under general circumstances. However, the efficient equilibrium does not necessarily satisfy elementary equity considerations, such as equal opportunities for households in a comparable situation.

The paper is concluded with an empirical investigation into the efficiency of the Dutch housing market.



1 Introduction

The housing market has been subject to severe government measures all over the world. Rent control has been enforced in some cities in the United States (with New York as the best known example), in all western European countries, in centrally planned economies and in many developing countries. Usually the purpose of rent control has been to provide some protection for the harsh forces of the market to people with low incomes. Economists have in general not been able to find many positive effects of the measures as the income redistribution effects of rent control in New York seem to have been negligible (see Gyourko and Linneman [1989], Linneman [1987]), Dutch rent control seems to have had severe negative consequences (Oosterhaven and Klunder [1988]), while in centrally planned economies black markets are flourishing (Alexeev [1988]). As an exception, Anas and Cho [1988] tend towards a positive overall evaluation of the Swedish system, but their conclusion - that the regulation of the Swedish housing market results in a Pareto improvement - is based on preliminary results only.

An immediate consequence of fixing the price of housing below the market level is excess-demand. Although this can in principle be avoided by subsidized building of new houses, in practice a situation of permanent discrepancies between demand and supply seems to have been the inevitable consequence of rent-control. Supplementary measures that distribute the vacant dwellings over the people who are willing to move are therefore necessary. The market becomes rationed.

In this paper we will study fixed price equilibria on the housing market. Such equilibria consist of rationing measures which ensure that effective (or rationed) demand does not exceed supply. We will be particularly interested in the possibilities to ration the market in an efficient way, in the sense that we will avoid all unnecessary restrictions and allow for all exchanges of dwellings that are mutually beneficial at the prevailing fixed prices. In this way we can reach Pareto-optimality. The fact that this kind of optimality is not (more or less) automatically ensured has to do with one special characteristic of the housing market : most of the dwellings are already occupied and the actions on the market are for a large part concerned with a redistribution of the given stock.

In the next section we will give a short review of some relevant literature. We then present our model (section 3), derive some preliminary results (section 4) and move on to the central propositions of the paper

(section 5). Possible extensions are discussed (section 6) and finally some conclusions will be formulated (section 7).

2 Review of some relevant literature

Since the mid-seventies there has been a great deal of interest in general equilibrium economics in so-called fixed price equilibria. (See e.g. Dreze [1974] and Benassy [1975] for some pioneering papers). The main motivation for this research has been the presumption that such equilibria could provide a micro-economic underpinning for Keynesian macro-economics (Malinvaud [1977]). Consequently, in this literature not much attention has been given to the study of such equilibria for specific markets for heterogeneous goods, such as the housing market, although it provides an important background to such research. Wiesmeth [1985] seems to be the only paper concerned with the housing market which is explicitly related to this literature, while Stahl and Alexeev [1985] develop a general equilibrium model for an economy with fixed prices and black markets which also seems to be relevant for further study of the housing market.

The econometrics of disequilibrium which have been developed in relation with the theoretical developments (see e.g. Maddala [1982]) mentioned above have been used for analysis of the housing market by Anas and Eum [1986].

Another important development at the general level is the interest in non-price control that arose in the same period and was pioneered by Kornai and his associates (see e.g. Kornai [1980], Kornai and Martos [1981]). The first theoretical studies of a housing market that is not equilibrated by prices seem to have been inspired mainly by this research (see Kornai and Weibull [1978], Snickars [1978] and Weibull [1983]) and were concerned with the derivation of a 'normal state' of the market. They referred more or less explicitly to the Swedish housing market, which is heavily regulated.

A third development that has to be mentioned here is the recent interest in the existence and uniqueness of price equilibria in markets where aggregate demand is determined on the basis of discrete choice models for individual decision-makers, usually called stochastic price equilibria (see Anas [1982], Eriksson [1986], Anas and Cho [1986], Smith [1988], Rouwendal [1990a]). Although the results that have been reached in this literature

are of a general nature, a main motivation for their derivation seems to have been their potential relevance for the study of the housing market. However, the housing market is very often a regulated market and is therefore of interest to look whether the same framework can be used to study markets which are not equilibrated by prices. This has indeed been done by Anas and Cho [1988] who try to develop an operational framework for the study of the heavily regulated Swedish housing market and by Rouwendal [1990b] where some existence theorems for rationed equilibria are proved.

The present paper is concerned with the existence of rationed equilibria that satisfy certain desirable properties. In particular, attention will be given to equilibria that are efficient in the sense that at the rationed equilibrium no mutually beneficial trades can be carried out at the prevailing prices. Optimality properties of rationing schemes were considered by Dreze and Mueller [1980], who introduced the term 'constrained Pareto optima'. The question we will be concerned with is whether there exist such constrained Pareto optima on the housing market, with its special characteristics of indivisibility and redistribution.

It should be stressed at once that a constrained Pareto optimum is not optimal in all respects : it refers to efficiency, not to equity (for a discussion of equity properties of rationing schemes see Sah [1987]). Nor does it give information about the desirability of fixing the prices (see Weitzman [1977]) or does it imply that there will be no reason for actors to engage in black market transactions. The analysis brings out clearly, however, that efficiency and equity considerations are not easy to reconcile.

3 Description of the model

In the model that will be employed behaviour of individual actors is the starting point of the analysis. This behaviour will be described by means of individual decision functions, which give the probability that an actor chooses a particular alternative. Although we do not use specific formulations of the individual choice functions, an almost natural interpretation is to view them as discrete choice models, such as the logit or probit model. This places the present paper in the line of research on stochastic equilibria.

We consider a heterogeneous population which consists of b actors. Each

actor belongs to one of M groups. The groups may have been distinguished on the basis of income, number of persons in the household or any other criterion judged to be relevant.

This population is distributed over N possible states, indexed $1, \dots, N$. Each of the states $1, \dots, N$ can be identified with a dwelling type. Since we study the distribution of a given population over a given stock of dwellings we will refer to our model as a closed model. The closed model provides a convenient starting point for the present analysis. However, since it is far more realistic to have the possibility to deal with households entering or leaving the market we will also discuss an open model in section 6.

It is assumed that the households are initially distributed over the existing housing stock in some exogeneous way. The probability that a household of group m ($m=1, \dots, M$) that is in state n will choose to move to state n' ($n, n'=1, \dots, N$) will be denoted as $\pi_{mn \rightarrow n'}$. This probability is assumed to be a function of the fixed prices p and of the rationing parameters Q , where p is an $(N+1)$ -dimensional vector and Q a $N \times N$ -dimensional matrix.

These rationing parameters will be denoted as $q_{n \rightarrow n'}$, and should be interpreted as the fraction of actors who are initially in state n and who intend to move to state n' that will be allowed to do so. We will not be concerned with the question how this fraction is composed. One interpretation is that people that have been waiting for the longest time are allowed to move first; then the expected waiting time equals $1/q_{n \rightarrow n'}$. Another is that the people who will be allowed to move are selected randomly, by giving them all the same probability $q_{n \rightarrow n'}$ of realizing their intention.

It will be assumed that all actors who wish to continue their sojourn in a particular state will always be able to do so, i.e. $q_{n \rightarrow n} = 1$ for all $n=1, \dots, N$. All realization fractions should have a value between 0 and 1, boundaries included. We will denote the set of matrices Q with elements $q_{n \rightarrow n'}$, that fulfill this criterion as Ω . This set is closed and convex.

In summary, we have :

$$\pi_{mn \rightarrow n'} = \pi_{mn \rightarrow n'}(p, Q), \quad (1)$$

$$m=1, \dots, M, \quad n, n'=1, \dots, N.$$

These choice probability functions will be assumed to be continuous in the realization probabilities $q_{n \rightarrow n'}$. We will make no further assumptions about the sign of the influence of these variables or differentiability.

The total number of actors that are initially in state n and are willing to move to alternative n' will be denoted as $D_{n \rightarrow n'}^*$, and is defined as :

$$D_{n \rightarrow n'}^*(p, Q) = \sum_{m=1}^M b_{mn} \cdot \pi_{mn \rightarrow n'}(p, Q), \quad (2)$$

$$n, n'=1, \dots, N.$$

It should be noted that this equation gives the number of intended moves, which should be distinguished from the number of realized moves, which will now be determined.

The total number of actors who intend to move from n to n' and are able to realize their intention will be denoted as $\hat{D}_{n \rightarrow n'}$, and is defined as :

$$\hat{D}_{n \rightarrow n'}(p, Q) = \sum_{m=1}^M q_{n \rightarrow n'} \cdot \pi_{mn \rightarrow n'}(p, Q), \quad (3)$$

$$n=1, \dots, N.$$

The number of people in state n who were willing to move to state n' , but were not able to realize their intention will be denoted as \tilde{D}_n and is defined as :

$$\tilde{D}_{n \rightarrow n'}(p, Q) = \sum_{m=1}^M (1 - q_{n \rightarrow n'}) \cdot \pi_{mn \rightarrow n'}(p, Q) \cdot b_{mn}, \quad (4)$$

$$n=1, \dots, N.$$

This definition is based on the assumption, introduced above, that actors will always be allowed to continue their initial situation if a desired move turns out to be impossible.

The total effective demand for a state n , to be denoted as D_n , is defined as the actual number of actors that will be in that state (voluntarily or involuntarily) at the end of the period and equals :

$$D_n(p, Q) = \sum_{n'=0}^N \{ \hat{D}_{n' \rightarrow n}(p, Q) + \bar{D}_{n \rightarrow n'}(p, Q) \}, \quad (5)$$

$n=1, \dots, N.$

Finally, we have to close our model by introducing supply. We will do this in a particularly simple way by assuming that the capacity of all states n , $n=1, \dots, N$, is fixed at a value S_n . When the model refers to the housing market this says that the number of dwellings of type n is fixed.

In the next section we will start our investigation of the possibilities to equilibrate demand and supply.

4 Some preliminary results

A weak definition of a rationed equilibrium is the following :

Definition 4.1 A rationed equilibrium is a set of realization probabilities

$$\{q_{n' \rightarrow n}^*, n, n'=0, 1, \dots, N\}, \text{ such that for all } n \in \{1, \dots, N\} D_n(p, Q^*) \leq S_n.$$

The definition simply requires that the rationing to be such that demand will never exceed supply and has been termed weak because it does not involve any requirement on the efficiency of the rationing scheme. The rationing may for instance be much too strong in that the effective demand D_n is smaller than the supply S_n while some realization probabilities $q_{n' \rightarrow n}$ are smaller than 1. In order to avoid such equilibria we will introduce a natural notion of efficiency, that will be termed 1-efficiency in order to distinguish it from others, to be introduced later on.

Definition 4.2 A rationed equilibrium is called 1-efficient if $D_n(p, Q^*) = S_n$ whenever $q_{n' \rightarrow n} < 1$ for some $n' \in \{0, 1, \dots, N\}$.

In a 1-efficient equilibrium it is impossible that an actor who intends to move to a dwelling of type n is able to find a vacant dwelling of that type.

We will now introduce a notion of equity in our rationing scheme that requires that all people willing to move to the same type of dwelling will

experience the same realization fractions, which are therefore independent of their initial situation. Formally we define :

Definition 4.3 A set of realization probabilities is uniform if $q_{n \rightarrow n'} = q_{n \rightarrow n''}$, for all $n, n' \in \{0, 1, \dots, N\}$.

Next, we make the following formal assumption :

Assumption 4.1 a) the choice probability functions $\pi_{mn \rightarrow n'}$, $m=1, \dots, M$, $n, n'=0, 1, \dots, N$, are continuous in the realization probabilities $q_{n'' \rightarrow n'}, n'', n'=0, 1, \dots, N$,
 b) $S_n \geq \sum_{m=1}^M b_{mn}$, $n=1, \dots, N$.

Part a) of this assumption is simply a repetition of the continuity requirement mentioned in section 3 above, part b) and is needed in order to ensure that our earlier mentioned assumption that all actors will be able to continue their initial situation is valid.

We have the following proposition :

Proposition 4.1 If assumption 4.1 is satisfied there exists a uniformly rationed equilibrium.

For the proof of this proposition we refer to Rouwendal [1990, proposition 8]. It consists of a simple fixed point argument.

It is remarkable that we can prove existence of a rationed equilibrium under such weak assumptions. This may be viewed as an affirmation of the intuitive notion that rationing can do the allocation job in cases where the price mechanism fails.

On the other hand it should be realized that the uniformly rationed equilibrium lacks some desirable properties which the price system possesses. The most important of these is that in a price equilibrium the actors are in an optimal situation in the sense that they cannot improve their position at the prevailing prices. In the rationed equilibrium there will be unsatisfied demand as long as there is one realization probability smaller than 1. In part, this unsatisfied demand is an inevitable consequence of the rationing. For another part, however, it may have to do

with the specific form of the rationing, even when it is 1-efficient.

To see this it suffices to observe that at the uniformly rationed equilibrium pairs of actors may be found that find it beneficial - at the prevailing fixed prices - to 'swap', that is, to exchange their dwellings. The uniformly rationed equilibrium can thus be inefficient in the sense that the unsatisfied demand is unnecessarily large. It is therefore natural to ask whether such inefficiency can be avoided.

In order to get an answer to this question we will leave the notion of a uniformly rationed equilibrium and allow the realization probabilities to be dependent on the original state of the actors. Furthermore, we will introduce the notion of 2-efficiency :

Definition 4.2 A rationed equilibrium will be called 2-efficient if no pair of actors can be found who are willing to exchange their dwellings.

An immediate consequence of this definition is that in a 2-efficient equilibrium any pair of realization probabilities $(q_{n \rightarrow n'}, q_{n' \rightarrow n})$ at least one element should be equal to 1. At least 50 % of all possible moves are allowed to take place unconstrained. Although this does not imply that at least 50 % of the actors will experience no constraints on their behaviour, it nevertheless indicates that regulation of the housing market does not automatically imply rationing of all possible moves.

The existence of a 2-efficient equilibrium will now be proven without any further assumptions :

Proposition 4.2 When assumption 4.1 is satisfied, there exists a 2-efficient rationed equilibrium.

Proof. Consider the following mapping :

$$f_{n \rightarrow n'}(Q) = \min(1, D_{n \rightarrow n'}^* / D_{n' \rightarrow n}^*), \quad (6)$$

$$n, n' = 1, \dots, N.$$

It is immediately clear from (6) that the functions $f_{n \rightarrow n'}$ are continuous in the variables $q_{n \rightarrow n'}$. Moreover, $f_{n \rightarrow n'}$ always takes on a value in the closed interval $[0, 1]$. The matrix-valued function $F(Q)$ is therefore continuous and

maps the closed and convex set Ω of all realization probabilities into itself. By Brouwer's theorem it may therefore be concluded that it has a fixed point Q^* . This fixed point corresponds with an equilibrium since $q_{n \rightarrow n'} \cdot D_{n \rightarrow n'} \leq q_{n' \rightarrow n} \cdot D_{n' \rightarrow n}$ for all $n, n' = 1, \dots, N$. \square

It would of course be desirable that the 2-efficient equilibrium is also 1-efficient. However, this will in general not necessarily be the case. To see this it should be observed that the notion of 2-efficiency is concerned with exchanges of dwellings only. If there are initially vacant dwellings, there can be a 2-efficient rationed equilibrium in which these vacant dwellings are not filled. Indeed, the fixed point that has been proven to exist refers to such a 1-inefficient equilibrium.

One may also wonder whether a 2-efficient equilibrium is uniform. Although the notions of 2-efficiency and uniformity do not exclude each other, they surely do not imply each other. There is no reason why at any given realization fractions the ratio's $D_{n' \rightarrow n} / D_{n \rightarrow n'}$ and $D_{n'' \rightarrow n} / D_{n \rightarrow n''}$ should be equal to each other. It must therefore be concluded that an equilibrium that is both 2-efficient and uniform will only occur by accident.

It is natural to ask whether there can also be 3 or higher efficient equilibria. The answer is affirmative. This will be shown in the next section, where we will prove our main results.

5 Efficient rationed equilibria

In the present section we will prove the existence of a rationed equilibrium that is simultaneously 1, 2, ..., N-efficient. For brevity, such an equilibrium will be called efficient. At such an equilibrium there are no additional transactions left which would be beneficial to all the actors involved. The efficient equilibrium can therefore be identified with a Pareto optimum.

It will be convenient to have a special term for equilibria that are 2, 3, ..., N-efficient, but not necessarily 1-efficient. Since the notion of 2, 3, ..., N-efficiency refers to exchanges of dwellings, in contrast to that of 1-efficiency, we will call such equilibria exchange efficient.

Before we prove our main propositions we first have to introduce some additional terminology. A loop will be defined as an ordered set of different states n , $n=1, \dots, N$ which has at least 2 elements. A loop $\{n_1, n_2, \dots, n_\ell\}$ should be identified with a closed sequence of connections $n_1 \rightarrow n_2 \rightarrow \dots \rightarrow n_\ell \rightarrow n_1$. It will be clear from this description that two loops in which the same elements appear in the same order, but with different starting elements are in fact identical. E.g. the loops $\{1, 2, 3\}$, $\{2, 3, 1\}$ and $\{3, 1, 2\}$ refer to the same closed sequence of connections. We will refer to an arbitrary loop by means of the symbol λ and denote the set of all loops as Λ . The maximum number of elements in a loop is clearly N .

It will be convenient to have an ordering of the elements of Λ . For this purpose we will use the convention that the smallest element of a loop will be put in the front position. A loop with a larger number of elements will be given a higher order. For loops with the same number of elements the one with the largest element in the front position will get the higher order. When both the number of elements and the element in the front position are equal the values of the elements in the second position are decisive, etc. The resulting ordering is : $\{1, 2\}$, $\{1, 3\}$, ... $\{1, N\}$, $\{2, 3\}$, $\{2, 4\}$, ... $\{N-1, N\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 2, \dots, N\}$.

In the previous section it has been shown that a 2-efficient rationed equilibrium can be identified with an equilibrium in which for all loops of length 2 at least one of the realization probabilities $q_{n_1 \rightarrow n_2}$ and $q_{n_2 \rightarrow n_1}$ equals 1. In the same way a K -efficient rationed equilibrium can be identified with a rationed equilibrium in which for each chain of length K at least one of the choice probabilities $q_{n_i \rightarrow n_{i+1}}$, $i=1, \dots, \ell$, with $n_{\ell+1}=n_1$, equals 1.

We are now in a position to prove our first main result :

Proposition 5.1 When assumption 4.1 is satisfied there exists an exchange efficient equilibrium.

Proof. We will make use of a fixed-point argument. For this purpose we will construct a mapping of the set Ω of all realization probabilities into itself. The construction of this mapping will be motivated by means of a distribution mechanism.

Consider a situation in which prices are fixed and realization fractions

are somehow determined. Since the realization fractions are given, the values of the variables $D_{n \rightarrow n'}^*$, can be determined. Now take the first loop of the set A , ordered in the way defined above, $\{1,2\}$, and diminish both $D_{1 \rightarrow 2}^*$ and $D_{2 \rightarrow 1}^*$ with $\min \{D_{1 \rightarrow 2}^*, D_{2 \rightarrow 1}^*\}$. Then move on to the second chain, follow the same procedure etc.

In general, when we arrive at the k -th loop we will determine the minimum of what is left of the demands $D_{n_i \rightarrow n_{i+1}}^*$, $i=1, \dots, l$, and subtract this. In this way we ensure that all exchange possibilities that exist will be used.

The value of $D_{n \rightarrow n'}^*$, that remains after all this procedure will be denoted as $\bar{D}_{n \rightarrow n'}$. It will be clear that $0 \leq \bar{D}_{n \rightarrow n'} \leq D_{n \rightarrow n'}^*$. Moreover, $\bar{D}_{n \rightarrow n'}$ is a continuous function of the $q_{n \rightarrow n'}$'s since every step in the determination of $\bar{D}_{n \rightarrow n'}$ is continuous.

Now define the following mapping :

$$f_{n \rightarrow n'}(Q) = 1 - \bar{D}_{n \rightarrow n'} / D_{n \rightarrow n'}^*, \quad (7)$$

$$n, n' = 0, 1, \dots, N.$$

The matrix-valued function F maps the closed, convex set Ω into itself and is continuous. We can therefore conclude, by Brouwer's theorem, that it has a fixed point. This point corresponds with an exchange efficient equilibrium since, by construction of the mapping, there are no additional exchange possibilities left. \square

An immediate consequence of this proposition is that an efficient equilibrium exists in markets where supply exactly meets demand :

$$\sum_{m=1}^M b_{mn} = S_n \quad (8)$$

$$n=1, \dots, N.$$

This special case is known as a balanced market and is repeatedly studied in the literature on stochastic price equilibria (see e.g. Eriksson [1986], Smith [1988]). This will be stated formally as :

Corrollary 5.1 In a balanced market there exists an efficient equilibrium.

In the general case, in which equation (8) is not valid for all n , we still have to prove existence of 1-efficiency in combination with exchange efficiency.

Proposition 5.2 When assumption 4.1 is satisfied there exists an efficient equilibrium.

Proof. As in the proof of proposition 5.1 we will construct a mapping of Ω into itself. Assume that realization fractions have been determined somehow. Start with the procedure that has been used for the proof of proposition 5.1. and determine the values of the variables $\tilde{D}_{n \rightarrow n'}$. The number of vacant dwellings of type n will be denoted as V_n .

Now consider the set of all ordered pairs (n_1, n_2) , $n_1 \neq n_2$ and order it as follows : $\{1,2\}, \{1,3\}, \dots, \{1,N\}, \{2,1\}, \{2,3\}, \dots, \{N,N-1\}$. Start with the first pair, $\{1,2\}$, and subtract $\min(\tilde{D}_{1 \rightarrow 2}, V_2)$ from $\tilde{D}_{1 \rightarrow 2}$ and add the same value to V_2 . Move on to the second pair, etc. Then start again with the the pair $\{1,2\}$ and repeat the procedure. Continue until either the number of actors intending to move to a state n or the number of vacancies of that state equals zero, for all $n=1, \dots, N$. Since the volume of the unsatisfied demand decreases during the procedure we can be sure that it will converge. The ultimate level of the unsatisfied demand will be denoted as $\tilde{\tilde{D}}_{n \rightarrow n'}$. This variable is continuous in the realization fractions $q_{n \rightarrow n'}$.

The mapping we use is analogous to the one defined above :

$$f_{n \rightarrow n'} = 1 - \tilde{\tilde{D}}_{n \rightarrow n'} / D_{n \rightarrow n'}, \quad (9)$$

$n, n' = 1, \dots, N.$

The existence of a fixed point is guaranteed in the usual way. This fixed point corresponds with an efficient equilibrium. \square

6 Discussion

In the present section we will discuss the contents of the propositions proved above and reflect on their implications for rationing policy.

We will start with an examination of the question whether the model can be extended to include the appearance of new households (by household

formation or immigration) and the departure of others (because of death or emigration). We can give an unconditional affirmative answer to this question. We will introduce a new state, 0 into the model. Actors who leave the market (for whatever reason) will be dealt with as moving to state 0, actors who enter the market as coming from state 0. There will of course be no capacity restriction for this state, but one might nevertheless introduce realization fraction $q_{n \rightarrow 0}$, reflecting e.g. limited emigration possibilities, if one wishes to do so. The number of actors of class m who are originally in state 0 will be denoted as b_{m0} and is assumed to be given. This extended model will be referred to as the open model.

The existence of an efficient equilibrium in the open model can be proven in the same way as was done for the closed model. Before one starts the procedure lined out in the proof of proposition 5.2 one can start with letting all those willing to move to state 0 and able to do so move out. After the procedure lined out in the proof of proposition 5.2 has been stopped, actors who are in state 0 can be allowed to move in the dwellings that still remain vacant. This implies that we have :

Proposition 6.1 When assumption 4.1 is satisfied there exists an efficient equilibrium in the open model.

We will give no detailed proof for this proposition, but the discussion that precedes its statement will suffice to give a clear idea how it can be obtained.

If one wishes to do so one can of course distinguish various substates in state 0. E.g. actors willing to enter the market may be distinguished in newly formed households in the geographical area to which the model refers and potential immigrants. Actors leaving the market may be distinguished in emigrants and deceased. The structure of the model remains essentially unchanged.

A second aspect of the model that needs some discussion are the equity properties of the mechanism that is used. It will be clear from section 4 that uniform rationing will in general not be efficient. Efficiency requires that the realization fractions can be differentiated on the basis of the initial situation of the actors. This implies that of two actors willing to move to the same type of dwelling one may have a much greater

probability of realizing his desire than the other, simply because he occupies a different type of dwelling.

Another equity aspect refers to the order in which the various existing loops are checked for the existence of mutually beneficial trades. Realisation of the trades associated with one loop may diminish, or even reduce to zero, trades associated with other loops, which are checked later on.

The problem becomes even more complex when we consider the open model. The procedure that has been outlined just before the statement of proposition 6.1 is highly beneficial to actors who are already participating in the market. Those entering the model will only be able to occupy dwellings that were not desired by anyone already participating in the market.

It is of course possible to change the procedure. The loops can be ordered in an arbitrarily chosen different way. Participants willing to enter the model can be dealt with alternatively by acting initially as if state 0 is a state like all others. After exchange efficiency has been reached, those willing to move to state 0 but thus far not able to do so will be moved out and the same procedure that was used in the proof of proposition 5.2 can be used to allocate the remaining vacant dwellings. In this way actors entering the model will in all probability have a better chance of obtaining a dwelling.

The choice for a particular rationing mechanism has therefore important implications for the equity of the resulting allocation, although it is not clear a priori which of the possible mechanisms that lead to an efficient equilibrium gives rise to the 'fairest' distribution of the housing stock over the population.

It will be clear from the preceding discussion that an efficient equilibrium will in general not be unique. Different allocation procedures will in general give rise to different equilibria which are all efficient. It might even be the case that the same allocation procedure does not correspond with a unique equilibrium.

The model presented here refers to the short run only. It is difficult to say what will happen in the long run. If unsatisfied demand will manifest itself in the same way in the next periods it may be expected that the

disequilibria concentrate at some bottlenecks in the market. It may happen that other parts of the market remain relatively unaffected, but it seems more likely that the excess demands spread and the market becomes 'stopped up'. Ultimately a number of demanders will reconcile themselves to their present situation, choose a second-best option or change their preferences, e.g. by reaching a different stage in their life cycle.

It will also be clear from the foregoing that most of the rationing schemes which are actually in use on regulated housing markets do not give priority to efficiency, but are more concerned with equity. E.g. the procedures that are used on the Dutch housing market - although far from transparent - give a priority treatment to special kinds of actors (i.e. differentiate on the basis of household characteristics) and use the 'first come, first served' rule. There seems to be no differentiation on the basis of the dwelling that will be left by a searching household. It may be expected therefore that the Dutch system of allocating vacant dwellings will not give rise to efficient outcomes. (This will be considered in more detail in the next section.) As the systems that are in use in other countries do seem to have the same characteristics, the same conjecture is also relevant there.

7 The Dutch Housing Market

The Dutch housing market consists of a heavily regulated part, containing rented dwellings, and a relatively free part, mainly consisting of owner-occupied dwellings. Ever since the Second World War prices have been determined by the government on the rented part of the market. For this empirical illustration we will therefore concentrate on the rationing that takes place in the housing market of the Dutch capital, Amsterdam. The data have been derived from the Housing Needs Survey (WoningBehoeftte Onderzoek) of 1981 and concern the households that intended to move from a rented dwelling to another rented dwelling within the Amsterdam area. We classified the dwellings on the basis of three criteria : single-family-dwelling or apartment, number of rooms and rent. Details are given in table 1.

Also in table 1 we listed the origins and destinations of the people intending to move to another rented dwelling within the Amsterdam area. It is clear from this table that the large majority of these people live in

Table 1 Intended moves

| Nr. | ap./s.f.d. | number of rooms rent | | number of dwellings | desired moves | |
|-------------------------|------------|------------------------------|------------|------------------------|----------------------|---------------------------|
| | | | | | number of origins | number of destinations |
| 1 | s.f.d. | 1,2,3 | ≤ 250 | 80 | 5 | 10 |
| 2 | s.f.d. | 1,2,3 | 250-450 | 30 | 1 | 31 |
| 3 | s.f.d. | 1,2,3 | ≥ 450 | 5 | 1 | 5 |
| 4 | s.f.d. | 4 | ≤ 250 | 15 | 8 | 2 |
| 5 | s.f.d. | 4 | 250-450 | 20 | 4 | 23 |
| 6 | s.f.d. | 4 | > 450 | 12 | 2 | 13 |
| 7 | s.f.d. | ≥ 5 | ≤ 450 | 143 | 1 | 62 |
| 8 | s.f.d. | ≥ 5 | > 450 | 25 | 1 | 20 |
| single family dwellings | | | | 330 | 23 | 166 |
| 9 | ap. | 1,2 | ≤ 250 | 504 | 99 | 27 |
| 10 | ap. | 1,2 | > 250 | 246 | 34 | 22 |
| 11 | ap. | 3 | ≤ 250 | 742 | 122 | 40 |
| 12 | ap. | 3 | 250-450 | 328 | 26 | 68 |
| 13 | ap. | 3 | > 450 | 112 | 17 | 16 |
| 14 | ap. | ≥ 4 | ≤ 250 | 418 | 70 | 24 |
| 15 | ap. | ≥ 4 | 250-450 | 555 | 40 | 69 |
| 16 | ap. | ≥ 4 | > 450 | 334 | 35 | 34 |
| apartments | | | | 3239 | 443 | 300 |
| total | | | | | 466 | 466 |

apartments and that many of them want to move to a single family dwelling. Supply and demand of the dwellings are therefore not in equilibrium. It is immediately clear from the table that of the 166 people intending to move to a single family dwelling, at most 23 can be served if we confine ourselves to moves within the existing stock of dwellings. At least 143 households will not be able to move, which implies that the average realization probability equals at most 0.69. Because of the heterogeneity within the groups of single-family-dwellings and apartments one may expect that the actual realization probabilities will even be lower.

It is clear from the figures in table 1 that there are large differences in the willingness to move between occupiers of the various housing types.

Table 2 Realized moves

| Nr. | within the existing stock | | to newly constructed dwellings | | starters | | migrants | |
|-----|---------------------------------|-------|--------------------------------------|-------|----------|-------|----------|-------|
| | or. | dest. | or. | dest. | new | ex. | new | ex. |
| 1 | 3.50 | 2.50 | 0.00 | 0.00 | 0.00 | 1.50 | 0.00 | 0.25 |
| 2 | 1.00 | 0.00 | 0.25 | 0.00 | 0.25 | 0.50 | 0.00 | 0.00 |
| 3 | 0.00 | 0.25 | 0.00 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 |
| 4 | 0.00 | 0.75 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 |
| 5 | 0.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 1.00 | 0.25 |
| 6 | 0.25 | 0.25 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 |
| 7 | 2.75 | 0.50 | 0.25 | 0.00 | 0.00 | 0.50 | 0.00 | 0.50 |
| 8 | 0.75 | 0.25 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 |
| sfd | 8.75 | 4.50 | 1.75 | 0.25 | 0.50 | 2.75 | 1.00 | 1.75 |
| 9 | 24.00 | 19.75 | 0.50 | 2.50 | 0.50 | 47.75 | 0.00 | 8.25 |
| 10 | 26.00 | 9.25 | 5.75 | 1.00 | 2.50 | 23.75 | 0.50 | 9.00 |
| 11 | 8.75 | 18.00 | 0.25 | 3.25 | 0.00 | 3.75 | 0.00 | 1.25 |
| 12 | 14.25 | 8.00 | 2.25 | 1.25 | 0.50 | 2.50 | 0.25 | 1.75 |
| 13 | 7.25 | 4.50 | 1.50 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 |
| 14 | 1.50 | 11.75 | 0.00 | 1.00 | 0.00 | 1.00 | 0.00 | 1.00 |
| 15 | 3.50 | 9.25 | 0.25 | 0.75 | 0.25 | 0.50 | 0.00 | 0.25 |
| 16 | 2.25 | 11.25 | 1.00 | 2.00 | 2.50 | 6.50 | 0.75 | 2.25 |
| ap. | 87.50 | 91.75 | 11.50 | 13.00 | 6.25 | 85.75 | 1.50 | 23.75 |
| to. | 96.25 | 96.25 | 13.25 | 13.25 | 6.75 | 88.50 | 2.50 | 25.50 |

When we restrict our attention to apartments (the small numbers of single family dwellings make it difficult to draw conclusions for this segment of the market) we get the strong impression that people in houses with a medium rent (between 250 and 450 guilders per month) are, in general, less willing to move than those in cheaper or more expensive houses. Also, it seems that houses with a medium rent are more in favour as an intended destination than those with a high or low rent. This makes the impression of significant disequilibria even stronger and enforces the conjecture mentioned above.

In the first two columns of table 2 the average yearly numbers of realized moves have been listed. If we make the assumption that the situation on the rented part of the housing market changes little over time, these figures can be regarded as an indication of the number of moves that will be realized. The ratio of this indicator and the number of intended moves gives an impression of the actual realization probabilities. Again disregarding the single-family-dwelling segment of the market, it can be verified that these realization probabilities show large differences and vary between 75 % for dwellings of type 10 and 2% for dwellings of type 14. The realization probabilities show a clear tendency to decrease with dwelling size (given the rent class) and are lowest for the low-rented dwellings (for all sizes of dwellings). This gives the impression that the rationing on the Amsterdam housing market is not uniform. (A thorough investigation of this conjecture would require a much more detailed analysis, however.) The average realization probability equals 0.21. If we also take into account the moves from the existing stock to newly constructed dwellings (see columns 3 and 4 of table 2), the average realization probability rises to 0.23.

In order to get some idea whether the actual rationing procedure on the Amsterdam housing market is efficient, the numbers of realizable moves within the existing housing stock have been computed with the aid of the procedure outlined in the preceding sections. It turned out that an efficient equilibrium was already reached when loops consisting of two elements were considered (i.e. no loops with a higher number of elements were possible after these small loops were considered). The results are shown in table 3.

It appears from that table that the number of realizable moves is larger than the average number of realized moves. The average realization probability equals 0.34. It should be stressed that this value has been reached by considering only (extended) exchanges of dwellings within the existing stock. Some of the actual moves have occurred because of the appearance of vacant dwellings because of migration, moves to special housing for the elderly, etc. Such moves have not been taken into account by our method.

In order to get an impression of the number of additional moves that would be made possible by vacancies that occur within the existing stock, we used the following procedure. We can compute the number of moves that

Table 3 Realizable Moves

| nr. | moves | vacancies | nr. | moves | vacancies |
|--------|-----------|--------------|------------|------------|--------------|
| 1 | 3 | 0.75 | 9 | 25 | 41.25 |
| 2 | 1 | -0.50 | 10 | 17 | 9.25 |
| 3 | 1 | 0.25 | 11 | 32 | 14.00 |
| 4 | 2 | 1.00 | 12 | 15 | -4.00 |
| 5 | 2 | -0.25 | 13 | 8 | -4.25 |
| 6 | 2 | 0.00 | 14 | 20 | 12.25 |
| 7 | 1 | -1.50 | 15 | 15 | 6.25 |
| 8 | 1 | -1.25 | 16 | 15 | 16.75 |
| s.f.d. | <u>13</u> | <u>-1.50</u> | apartments | <u>147</u> | <u>91.50</u> |

had an existing (i.e. non-newly-constructed) dwelling as their destination by adding up the number of destinations of moves within the existing stock, the number of starters and migrants that moved into such a dwelling. When one subtracts from this total the numbers of origins of the moves within the existing dwelling stock, one gets an estimate of the number of dwellings within the existing stock that become vacant. These numbers are also listed in table 3. From these figures one gets the impression that a substantial number of additional moves would be possible if these vacancies were also taken into account. However, it turns out that a large number of the vacancies occur for dwelling types which are not desired by many households as a destination of their moves. If : a) we regard a negative number as an indication of a diminishing stock of the relevant dwelling type and b) take the minimum of the number of estimated vacancies and the number of desired destinations of households that have not yet moved as an indication of the number of additional moves, we arrive at a total number of additional possible moves of 31.25. This would increase the average realization probability to 0.41, and even to 0.43 if also moves to newly constructed dwellings are taken into account. This is significantly higher than the actual figure.

Should one conclude from the figures presented above that the actual rationing system of the Amsterdam area is inefficient ? One is tempted to give a positive answer. An efficient rationing procedure may be too much

to expect from a local authority if the loops that are involved are large. In the computation of the realizable moves presented in figure 2, however, there occurred no moves containing more than two elements. The computation of the additional moves, made possible by the occurrence of vacancies, may be characterized as conservative, since it does not consider the possibility of additional moves within the existing stock which become possible as a result of moves towards these vacancies. It may be added that we have not considered the possibility of second-best choices which occur when households give up some of their desires. This phenomenon may be expected to be of some importance in a heavily rationed housing market.

However, it should also be realized that the information about the desired dwelling type refers to three aspects only (sfd/apartment, number of rooms and rent), while in reality there may be number of other characteristics which are relevant, such as the location within the municipality. So our evidence is not completely conclusive.

8 Conclusion

In this paper it has been shown that in a rationed housing market there may exist unnecessary inefficiencies which can be removed by examination of the possibilities for mutual exchange. The almost inevitable result is that realization probabilities will vary with the original situation of the household and that the possibilities to make these probabilities dependent on other criteria diminishes. Equity and efficiency - again - turn out to be hard to reconcile. The paper is concluded with an empirical investigation into the Amsterdam housing market. The results indicate that the prevailing rationing mechanism on that housing market does not lead to efficient outcomes.

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